



TOPOLOGY OPTIMIZATION TECHNIQUES

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CONTENTS

- Topology optimization
- Topology optimization techniques
- Overhangs
- Bridging and cavities in design
- Key characteristics and considerations in topology optimization
- Topology optimization under material uncertainty and manufacturability
- Topology optimization problem
- Case study

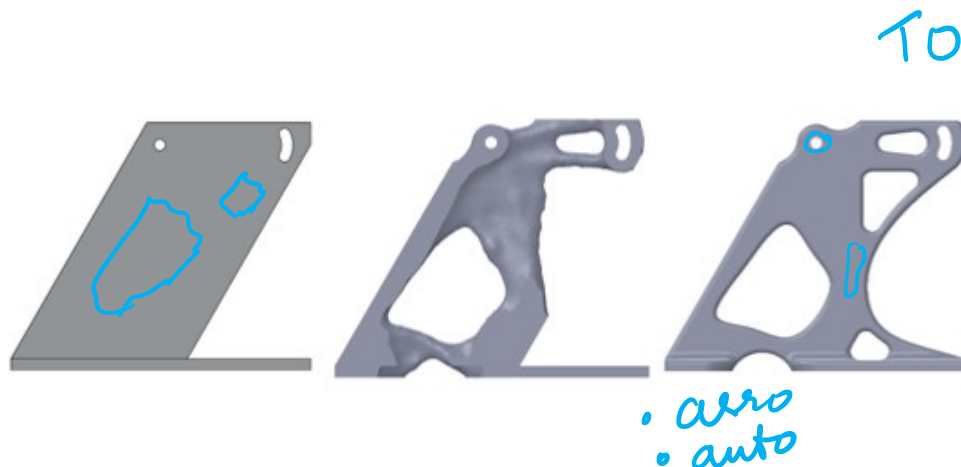
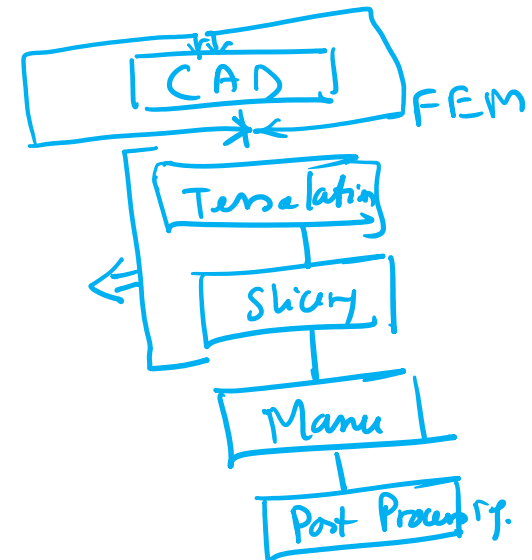


TOPOLOGY OPTIMIZATION

- constraints
- objective function

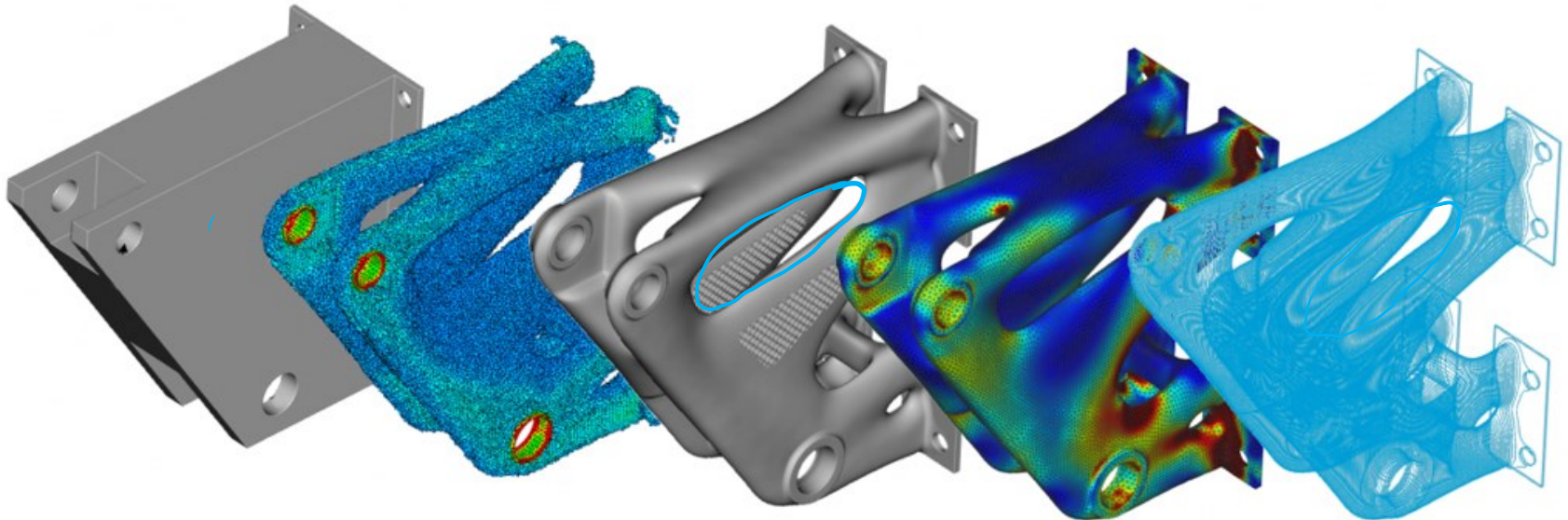
WHAT IS TOPOLOGY OPTIMIZATION?

Topology optimization is a mathematical method that spatially optimizes the distribution of material within a defined domain, by fulfilling given constraints previously established and minimizing a predefined cost function



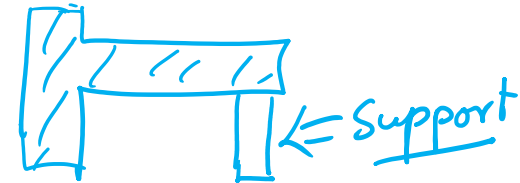
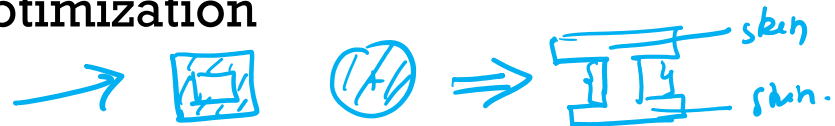
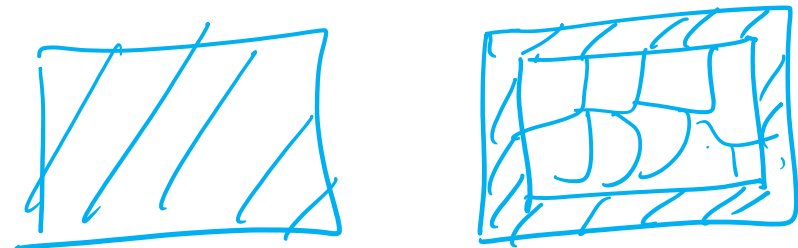
KEY ELEMENTS

1. The initial part should be larger than the final desired component .
2. Stress in the initial part should not be too high
3. Understand manufacturing limitations



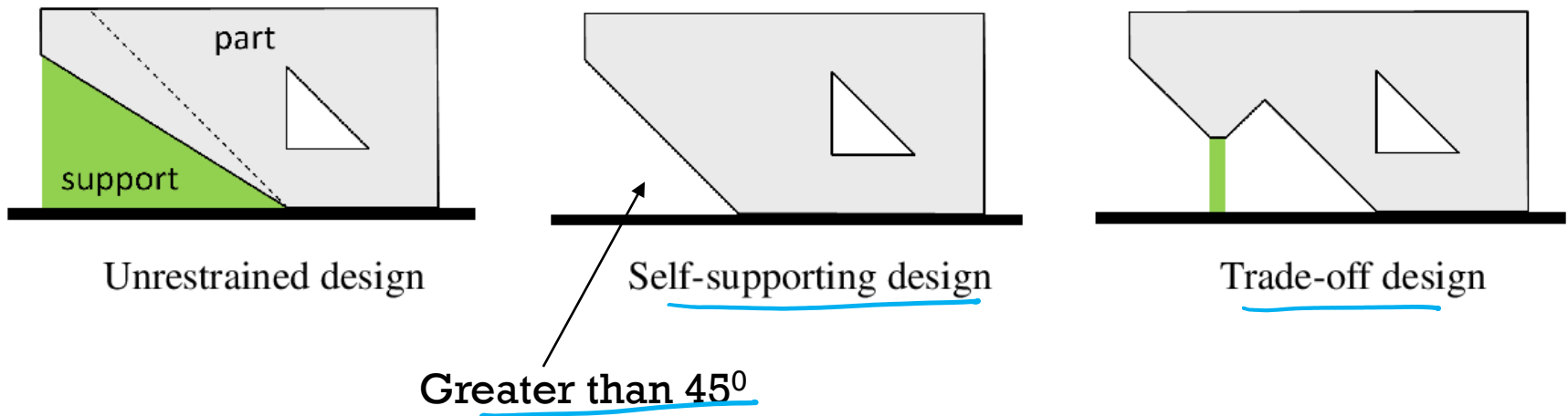
TOPOLOGY OPTIMIZATION TECHNIQUES

1. Support structure design
2. Support slimming
3. Overhang-free topology optimization
4. Porous infill optimization
5. Lattice material optimization (meta-material optimization)



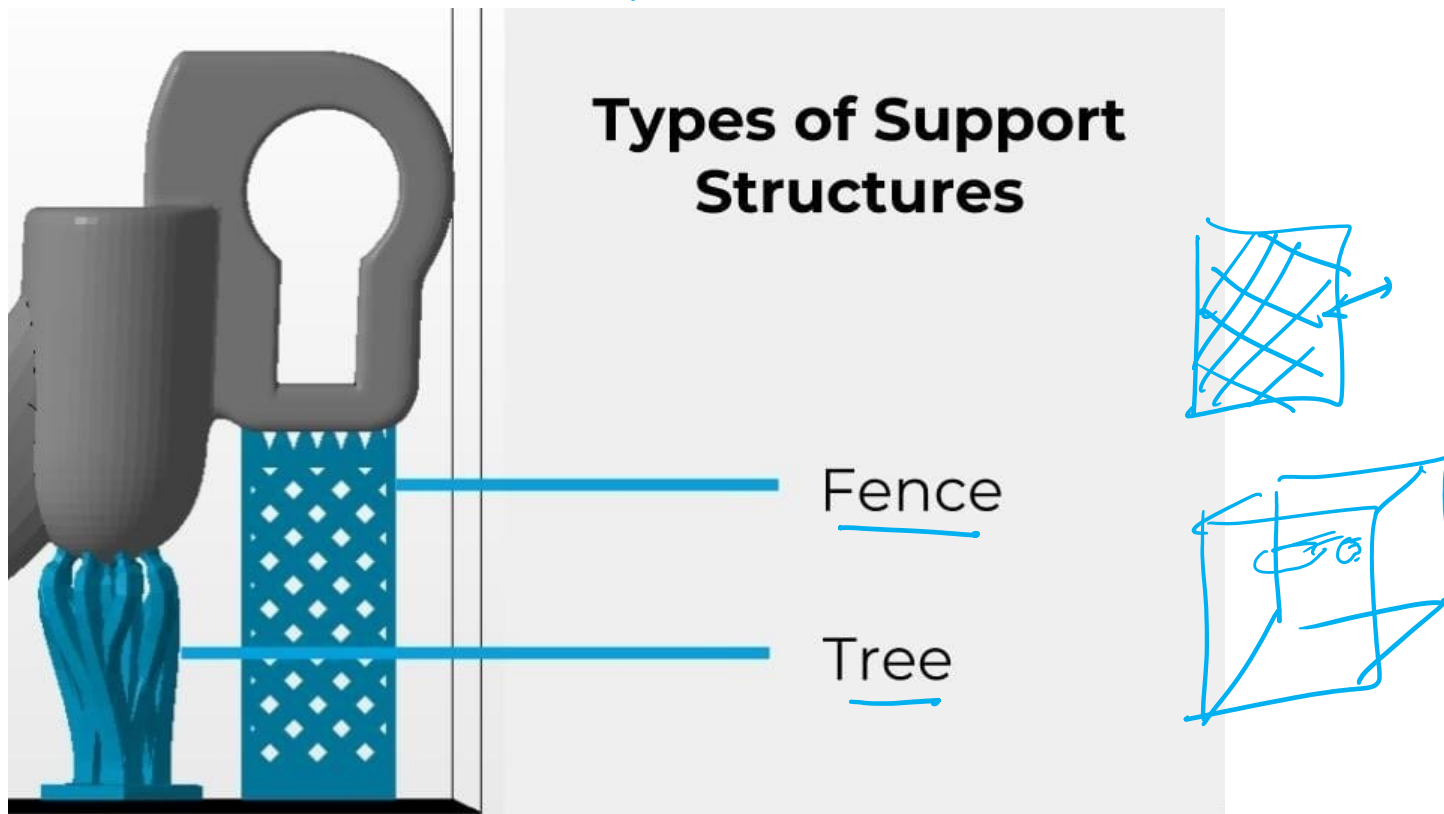
TOPOLOGY OPTIMIZATION TECHNIQUES

1. SUPPORT STRUCTURE DESIGN



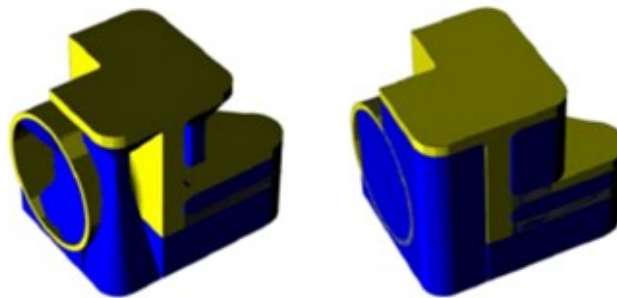
TOPOLOGY OPTIMIZATION TECHNIQUES

1. SUPPORT STRUCTURE DESIGN

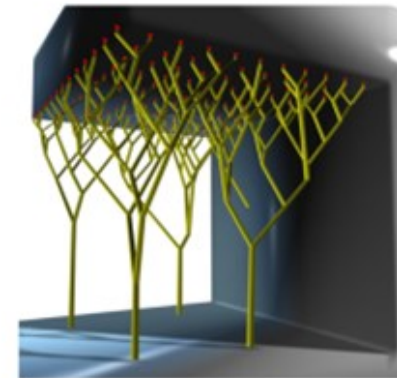


TOPOLOGY OPTIMIZATION TECHNIQUES

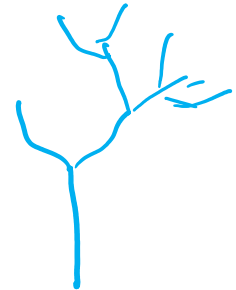
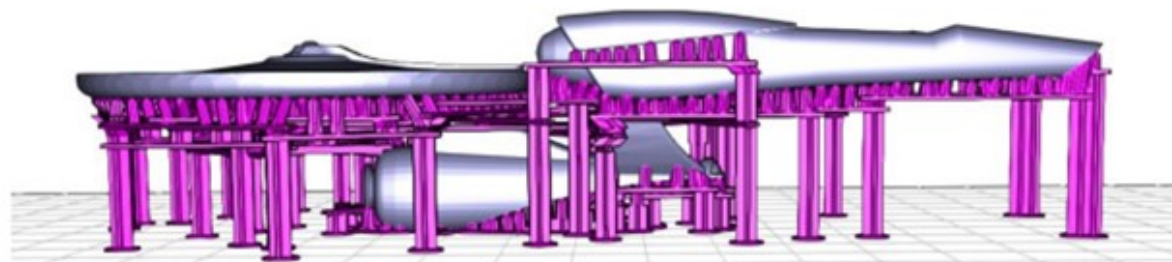
2. SLIMMED SUPPORT PATTERNS



Comparing slope and straight wall supports



Tree-like support

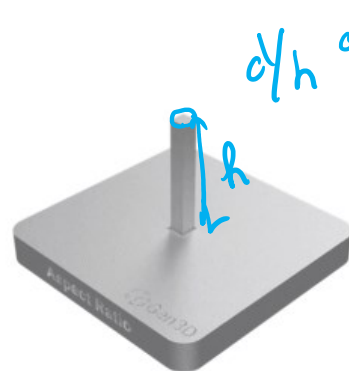


Bridge-like support

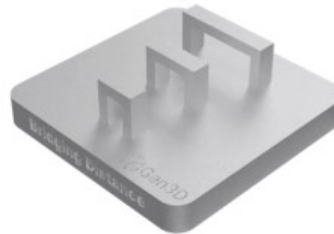


TOPOLOGY OPTIMIZATION TECHNIQUES

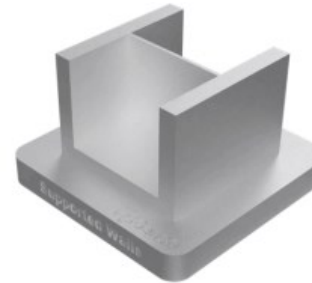
3. OVERHANG-FREE TOPOLOGY OPTIMIZATION



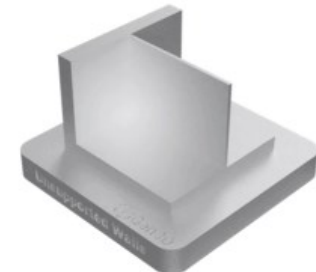
Max aspect ratio



Max bridging distance



Min supported walls



Min unsupported walls



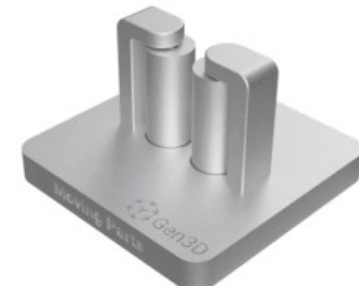
Min vertical hole



Max horizontal hole



Max overhang angle



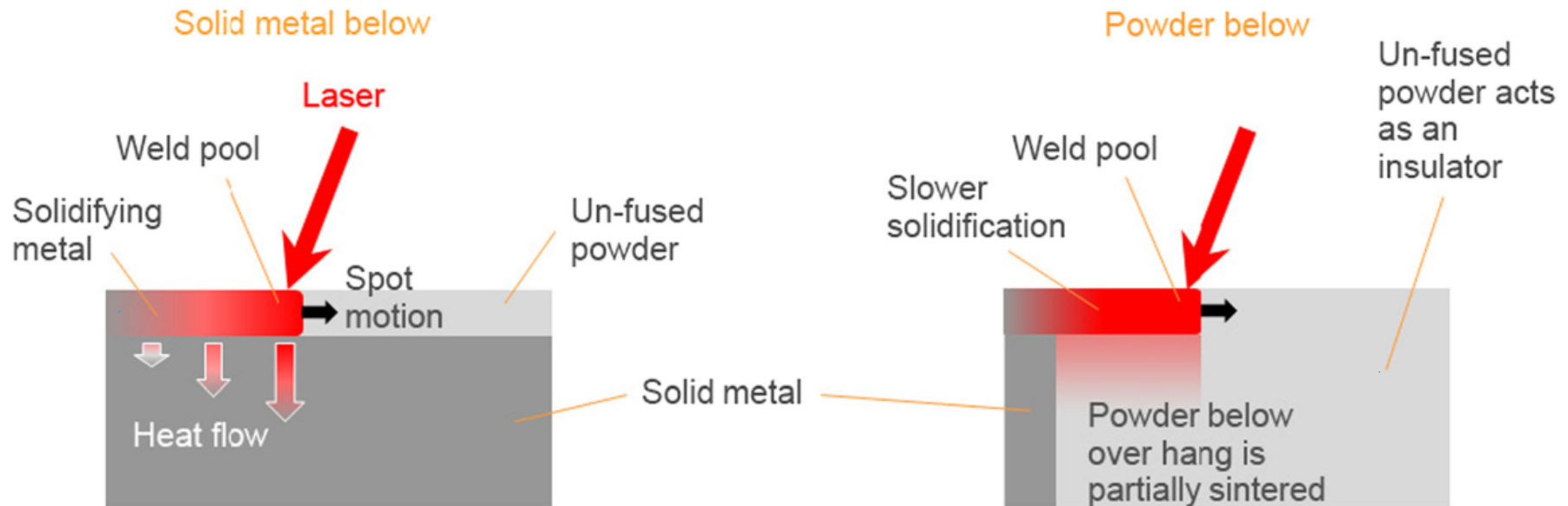
Moving part tolerance



TOPOLOGY OPTIMIZATION TECHNIQUES

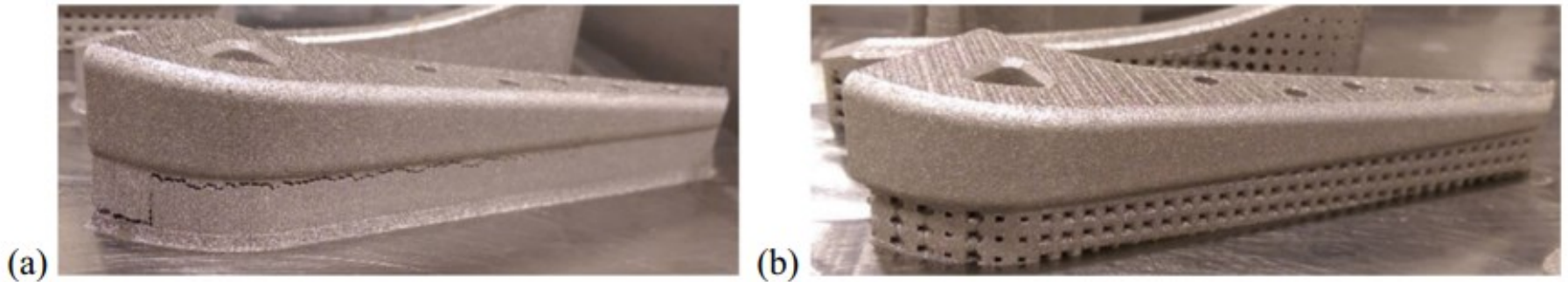
3. OVERHANG-FREE TOPOLOGY OPTIMIZATION

- Overhanging geometries created with PBF AM technologies
 - Loose powder act as support material



TOPOLOGY OPTIMIZATION TECHNIQUES

4. POROUS INFILL OPTIMIZATION

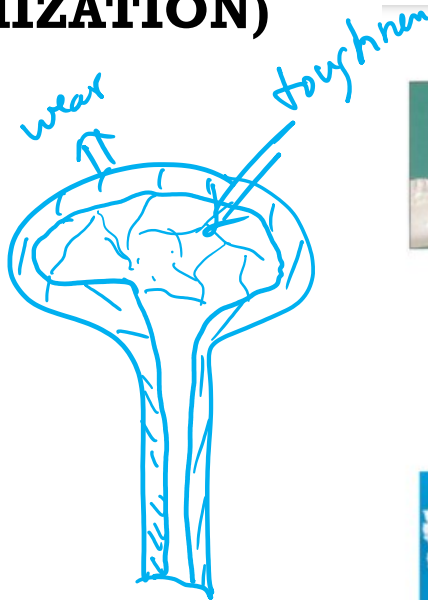
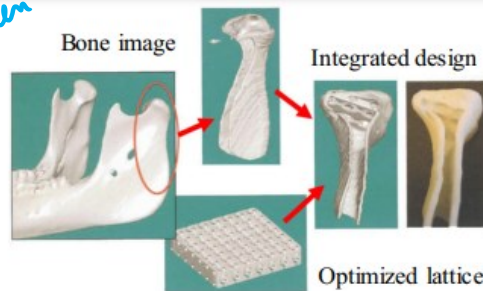


- a) Part built with a generic, un-optimized support has cracks, and
- b) Part built with an optimized support structure remains intact

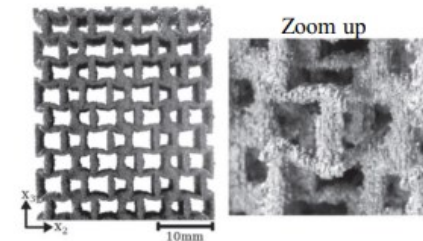


TOPOLOGY OPTIMIZATION TECHNIQUES

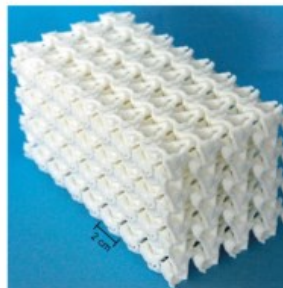
5. LATTICE MATERIAL OPTIMIZATION (META-MATERIAL OPTIMIZATION)



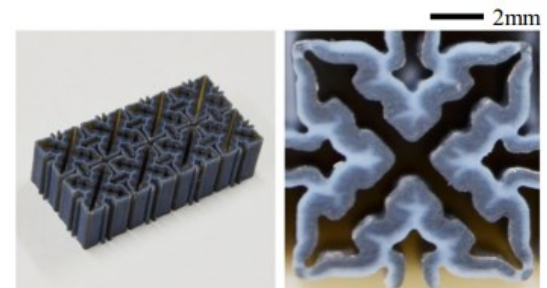
(a) Tissue engineering scaffold lattice design (Hollister 2005)



(b) Negative Poisson's ratio lattice by electron beam melting (Schwerdtfeger et al. 2011)



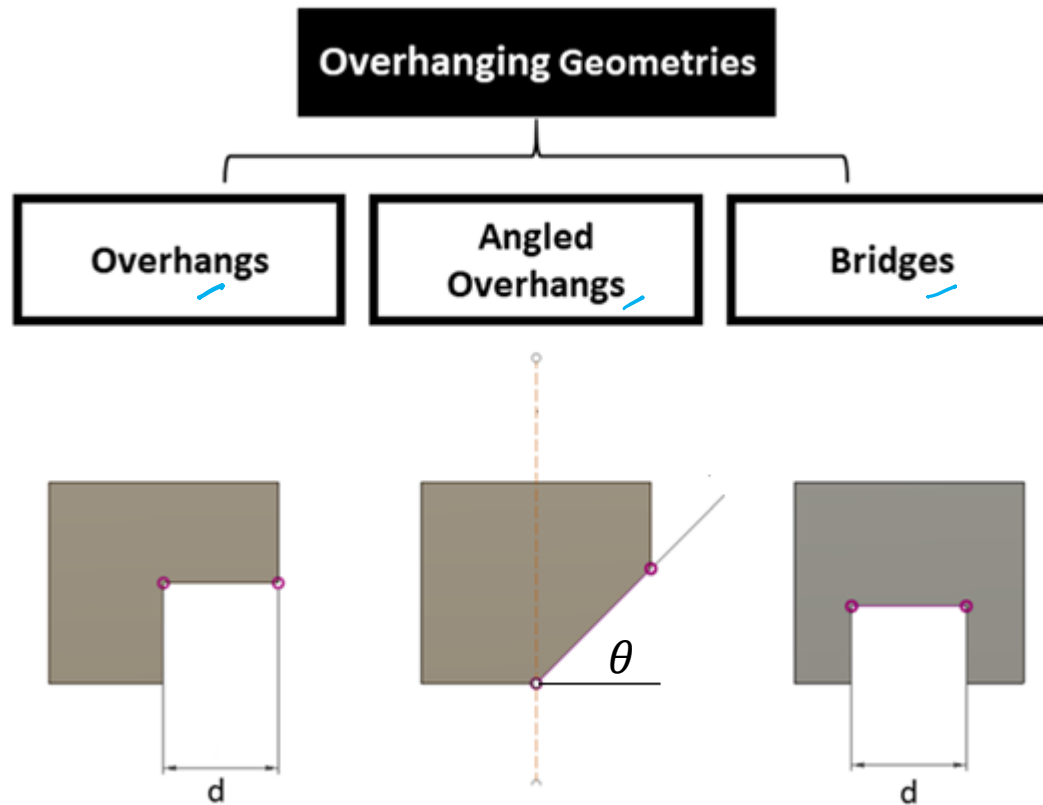
(c) Negative Poisson's ratio lattice by Nylon selective laser melting (Andreassen et al. 2014)



(d) Negative thermal expansion lattice by photopolymer multi-material AM Takezawa et al. 2015

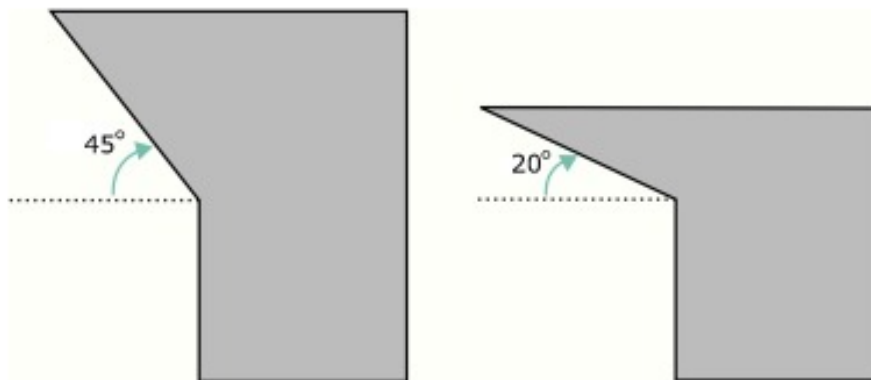


OVERHANG GEOMETRY

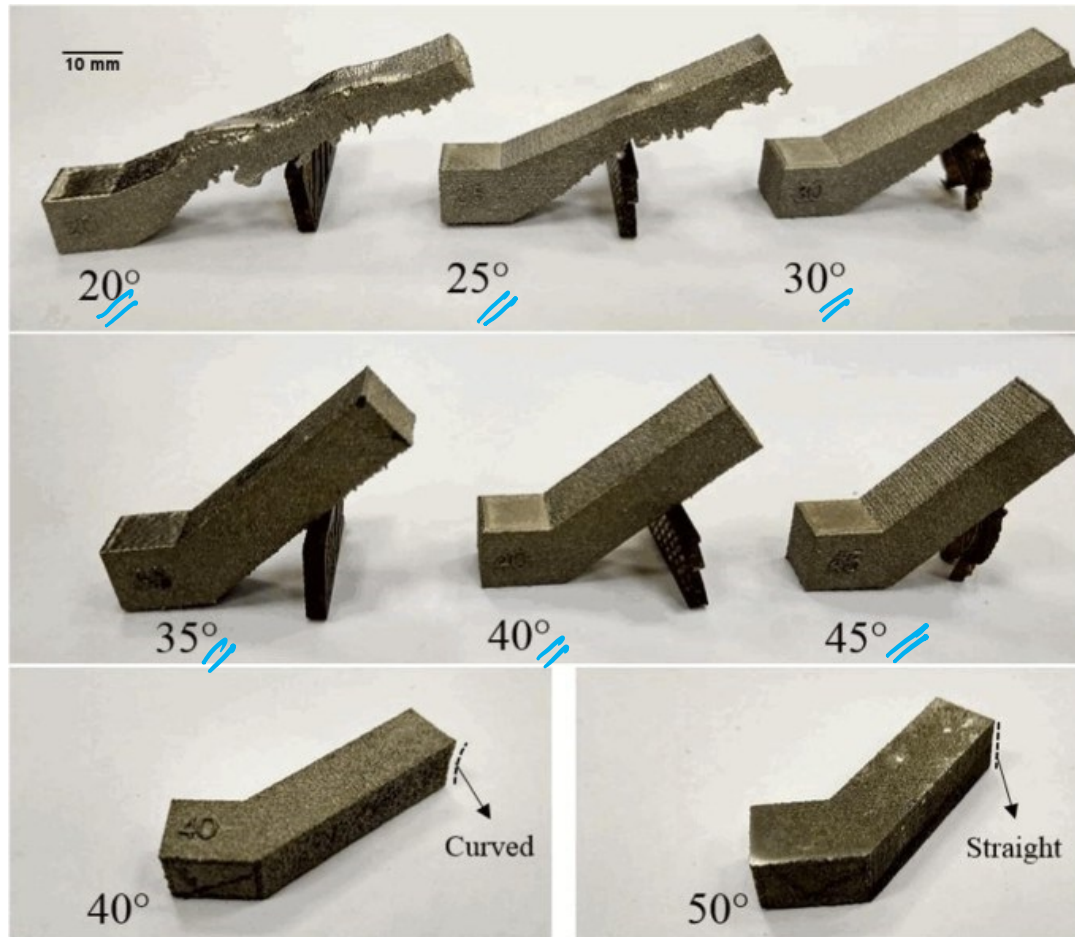


OVERHANG CONTROL

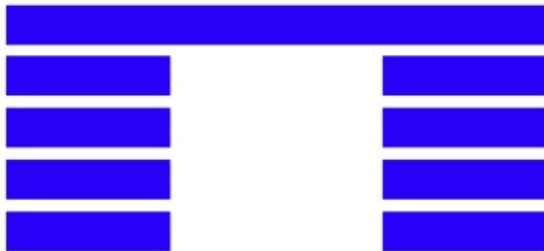
- Parts with an overhang angle less than 45° from the horizontal build surface plane typically cannot be printed.



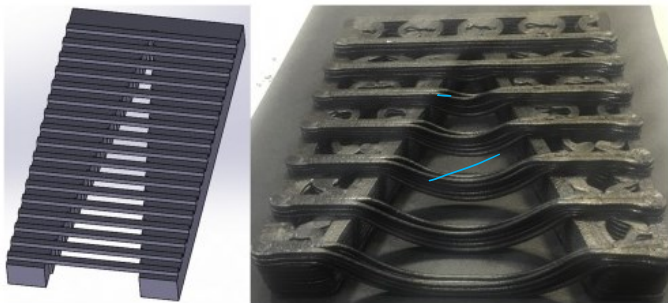
OVERHANG CONTROL



BRIDGING



- Bridges are connected to the body of the part at both ends.

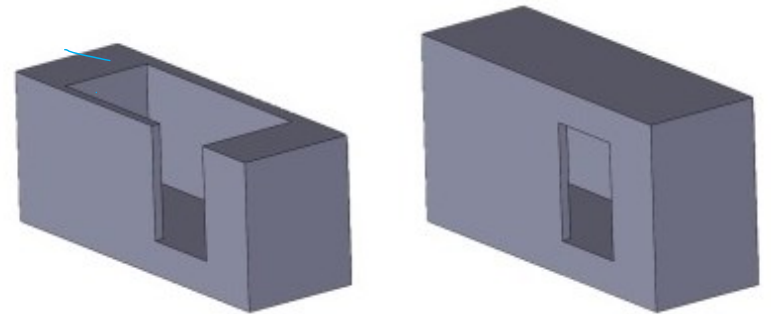


- To test the maximum bridging distance, a part is designed with a V-shaped cut on the bottom surface
- A failure is determined as a bead that breaks off or sags down more than $\frac{1}{2}$ of a layer height.



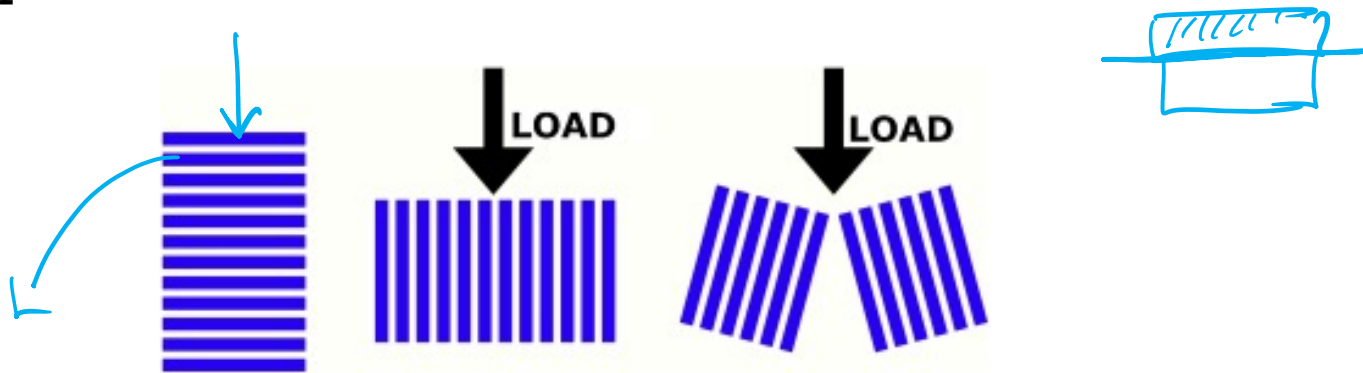
CAVITIES

- Whether a cavity is printable depends on the geometry and design of the part.
- Its geometry must be self-supporting to avoid the requirement of support material
- An opening must be left in the cavity so that the support material can be removed



IMPROVING STRENGTH IN THE Z-DIRECTION

- The bond between a molten layer and a solidified layer is much weaker than the bond between molten beads within the same layer



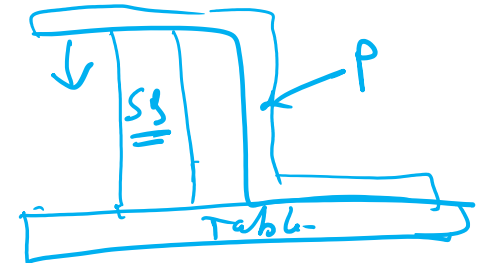
A printed part with layers stacked vertically (left), a load applied in line with the layer bonds (center), and the result of the load (right)



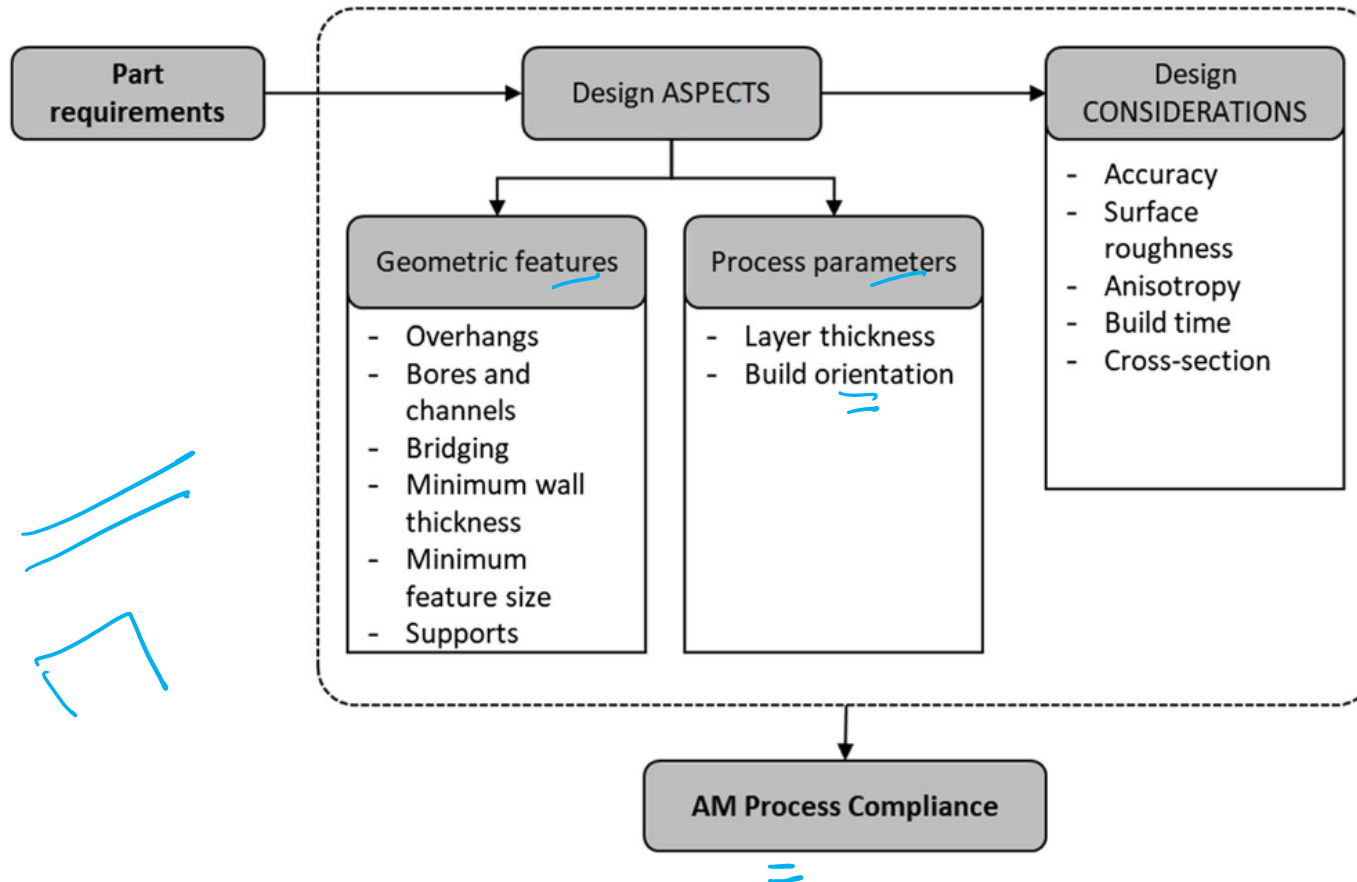
TOPOLOGY OPTIMIZATION

KEY CHARACTERISTICS

<u>Process-specific characteristics</u>	<u>Component-specific characteristics</u>
•Material	•Geometry
•Component orientation	•Dimensional accuracy
•Fiber orientation	•Roughness
•Build strategy	•Density
•Support strategy	•Strength
•Layer thickness	•Stiffness
•Nozzle/beam diameter	•Color
•Temperature/power	•Transparency



CONSIDERATIONS IN DESIGN (TO)



TOPOLOGY OPTIMIZATION UNDER MATERIAL UNCERTAINTY

- Specimens are usually built-in various orientations.
- Once the material properties of concern are quantified, a mean and standard deviation can be calculated for each property.
- Finally, this material property information can be fed back into the topology optimization algorithms resulting in more robust and reliable solutions

$$\text{Tensile Strength} = X \pm$$

$$\text{Shear} \quad \quad \quad = Y \pm$$

$$\text{Particle Size} \quad \quad = Z \pm$$

$$\text{Particle Shape} \quad = I \pm$$



TOPOLOGY OPTIMIZATION UNDER MANUFACTURING UNCERTAINTY

- Manufacturing process will always deviate from the design, even if that deviation is minimal.
- There is a need to develop approaches to tackle material shrinkage during the AM build.
- Significant residual stress buildup during the build.
- Significant “spring back” distortions while removing parts from the build platform.
- In many cases, topology optimized geometries are inaccessible for post-machining

$$\begin{aligned} \text{hole} &= 6 \text{ mm (design)} \\ &= 6 \pm 0.1 \text{ mm (manuf)} \end{aligned}$$



LAYER TIME LIMITATIONS

$T_0 \rightarrow$ design
 $T_0 \rightarrow$ design +
Manufacturing.

- The thermal properties of a material dictate layer time limitations and some aspects of design
- Can lead to thermal stresses and geometric distortions.

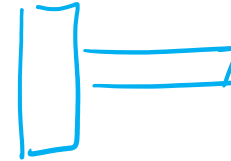


Feed rates $\rightarrow \uparrow$
need a time for layer cooling.

Layers drooping and collapsing caused by insufficient layer cooling time.

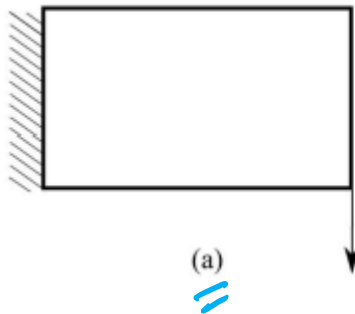
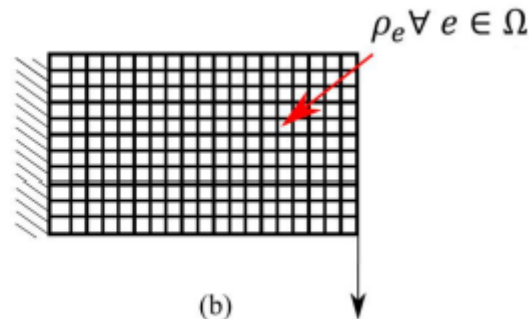
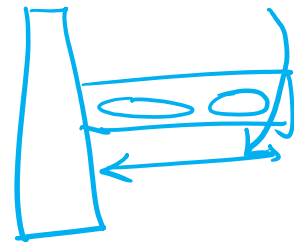


TOPOLOGY OPTIMIZATION PROBLEM FORMULATION



■ 2D EXAMPLE FOR TO OF A CANTILEVER BEAM

- a) Design domain Ω with kinematic loading conditions, i.e. point load at lower right vertex, while left edge remains fixed.
 - b) Discretised domain (schematic, the actual mesh is finer) with each finite element assigned a design variable ρ_e (subscript 'e' denotes the element number).
- **Aim is to find the optimal layout with densities that are either zero or one, such that the structure has maximum stiffness**



TOPOLOGY OPTIMIZATION

PROBLEM FORMULATION

- Solid Isotropic Material With Penalization (SIMP) approach in combination with finite element analysis (FEA) is used for numerical computation of the optimization objective and constraints.

$$\min : c(\rho) = F^T U = U^T K U = \sum_{e=1}^N (\rho_e)^p \mathbf{u}_e^T \mathbf{k}_e \mathbf{u}_e, \quad (1)$$

Which is subjected to $\mathbf{K} \mathbf{U} = \mathbf{F},$ (2)

Equilibrium equation for the discretized FE problem is given by equation (2)

The SIMP penalization is imposed as a power law on the density values given by the penalty factor $p > 1$, as shown in equation (1)

- ρ is the vector of all the design variables ρ_e ,
- $\mathbf{U}$ and $\mathbf{F}$ are the nodal degrees of freedom and load vectors respectively,
- $\mathbf{K}$ is the global stiffness matrix,
- $\mathbf{u}_e$ and $\mathbf{k}_e$ are the element displacement vector and stiffness matrix respectively,
- N is the total number of elements,



TOPOLOGY OPTIMIZATION

PROBLEM FORMULATION

- Next step is the calculation of sensitivities i.e. the design derivatives of objective c and constraint $g(\rho)$ with respect to design variable ρ_e

$$g(\rho) = \frac{V(\rho)}{V_0} - \bar{f} \leq 0, \quad (3)$$

$$0 < \rho_{\min} \leq \rho_e \leq 1, \quad \forall e \in \Omega, \quad (4)$$

- $\rho_{\min}$ is the minimum physical density (non-zero to avoid singularity of K),
- V and V_0 are the volume of solid region and design domain respectively
- $\bar{f}$ is the prescribed maximum volume fraction of material allowed in the design domain.

The design derivative of Equation (3) can be easily found as it depends directly on design variable ρ_e , but for Equation (1), the derivatives of the nodal degrees of freedom with respect to the design variables are required when direct differentiation is applied



TOPOLOGY OPTIMIZATION

PROBLEM FORMULATION

- There are many design variables and a comparatively small number of response functions (i.e. objectives and constraints)
- Hence avoid explicit computation of the derivatives of the nodal degrees of freedom for computational tractability
- The adjoint method is used, where an augmented compliance response is defined by adding the governing equation (Equation 2) in zero form, multiplied with a Lagrange multiplication

$$\underline{c = F^T U + \lambda^T (F - KU)} \quad (5)$$



TOPOLOGY OPTIMIZATION

PROBLEM FORMULATION

- Next, Equation (5) is differentiated with respect to the design variable ρ_e and the terms containing the derivative with respect to displacements are collected

$$\frac{\partial c}{\partial \rho_e} = (\mathbf{F}^T - \lambda^T \mathbf{K}) \frac{\partial \mathbf{U}}{\partial \rho_e} - \lambda^T \frac{\partial \mathbf{K}}{\partial \rho_e} \mathbf{U}. \quad (6)$$

- Equation (6) can be simplified to

$$\frac{\partial c}{\partial \rho_e} = -\lambda^T \frac{\partial \mathbf{K}}{\partial \rho_e} \mathbf{U}, \quad (7)$$

- such that λ satisfies the adjoint equation

$$\mathbf{F}^T - \lambda^T \mathbf{K} = 0. \quad (8)$$

KU

Equation (8) resembles the form of equilibrium Equation(2)



TOPOLOGY OPTIMIZATION PROBLEM FORMULATION

- For the choice $\lambda = U$, the displacement derivatives vanish from Equation (6)
- Hence, the sensitivity of the objective with respect to the design variable can be efficiently evaluated using

$$\frac{\partial c}{\partial \rho_e} = -U^T \frac{\partial K}{\partial \rho_e} U$$

- The final solution to the problem is shown in figure given below

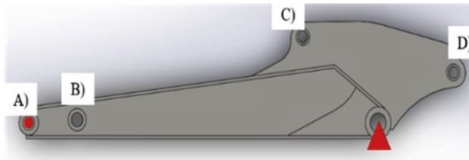
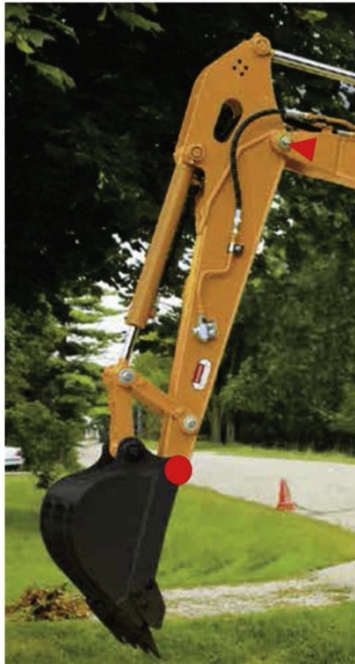
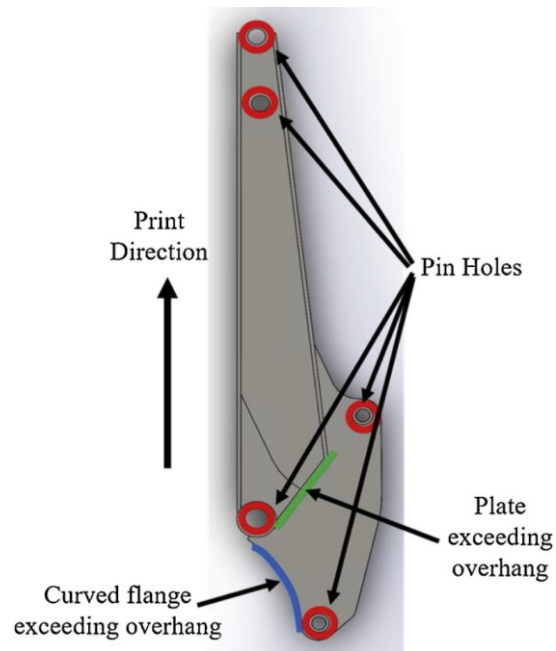


TOPOLOGY OPTIMIZATION PROBLEM FORMULATION

- Typically, an additional linear adjoint equation has to be solved to compute the design derivative for each response.
- Also, note that design independent load is assumed here
- Hence, the derivative of F with respect to the design variable ρ_e vanishes, which might not be the case for other more complex problems.



CASE STUDY EXAMPLE

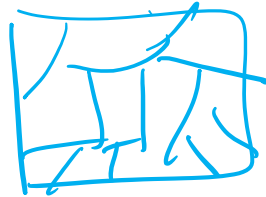


Modified Geometry



CHALLENGES IN TOPOLOGY OPTIMIZATION

- Effective properties prediction of the lattice structure
- Material anisotropy in additive manufacturing
- Structural fatigue performance in additive manufacturing
- Design and manufacturing of functionally graded material



POINTS TO PONDER

- What is topology optimization?
- Enlist key elements of topology optimization?
- Name some techniques for topology optimization.
- Explain the features in topology optimization such as overhangs, bridges and cavities.
- What major considerations are taken in design?
- Write a note on topology optimization for material uncertainties and manufacturability.



ASSIGNMENT

- Design a topology-optimized bracket for aerospace application and show the difference from initial to the final design.
- List major benefits of design.



THANKYOU

